EX. 1

DSB-SC $\longrightarrow$ NO CARRIER $\Rightarrow$

$f_c = 800\,KHZ \quad A_c = 100$

$\Uparrow$

$\Rightarrow$ MODULATED SIGNAL $v(t) = m(t) \cdot c(t)$

(a) $\times$ $\underline{\hat{v}(t)} = a(t) \sin 2\pi f_c t + b(t) \cos 2\pi f_c t$

$v(t) = a(t) \cos 2\pi f_c t - b(t) \sin 2\pi f_c t =$

$100 \cos 2\pi f_c t \left(\cos 2000\pi t + 2 \sin 2000\pi t \right)$

$\Downarrow$

$a(t) = 100 \left(\cos 2000\pi t + 2 \sin 2000\pi t \right)$

$b(t) = \phi$

$\Downarrow$

$\hat{v}(t) = 100 \sin 2\pi f_c t \left(\cos 2000\pi t + 2 \sin 2000\pi t \right)$

(b) $v(t) = 100 \cos 2\pi f_c t \, (\cos 2\pi f_a t + 2 \sin 2\pi f_a t)$

DSB-SC

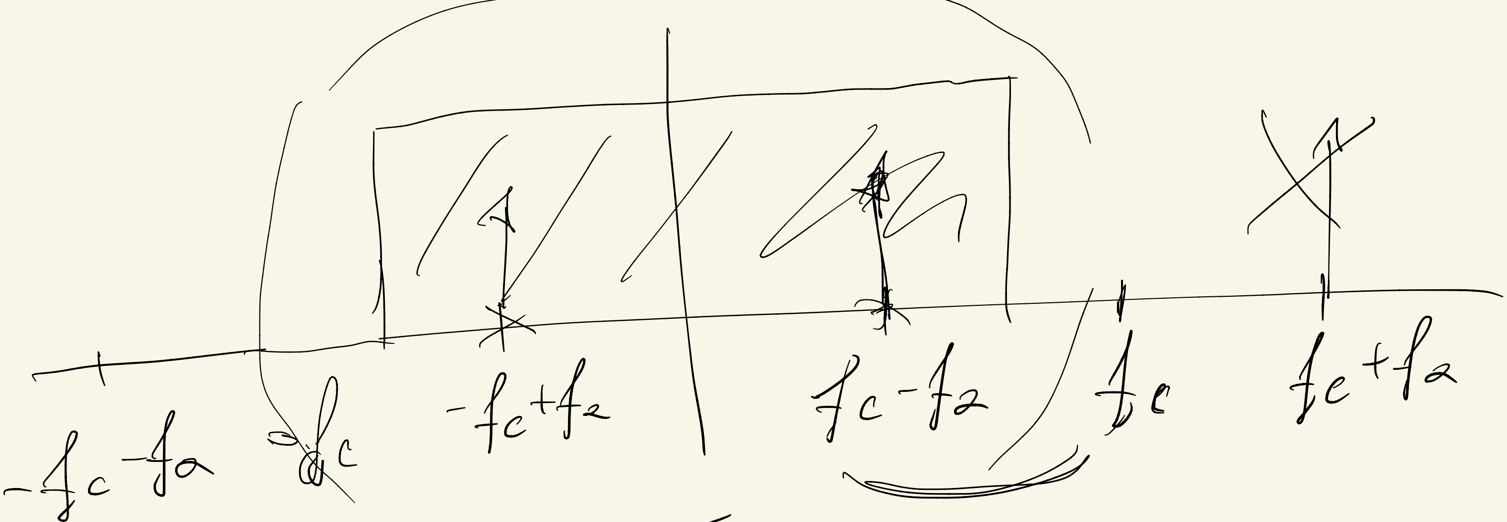
$$f_a = 1000$$

$$V(f) = 50[\delta(f-f_c) + \delta(f+f_c)] *$$

$$\frac{1}{2}\left[\delta(f-f_a) + \delta(f+f_a) + \frac{2}{j}\delta(f-f_a) - \frac{2}{j}\delta(f+f_a)\right]$$

$$= 25[\delta(f-f_c-f_a) + \delta(f-f_c+f_a) - 2j\,\delta(f-f_c-f_a) + 2j\,\delta(f-f_c+f_a)] +$$

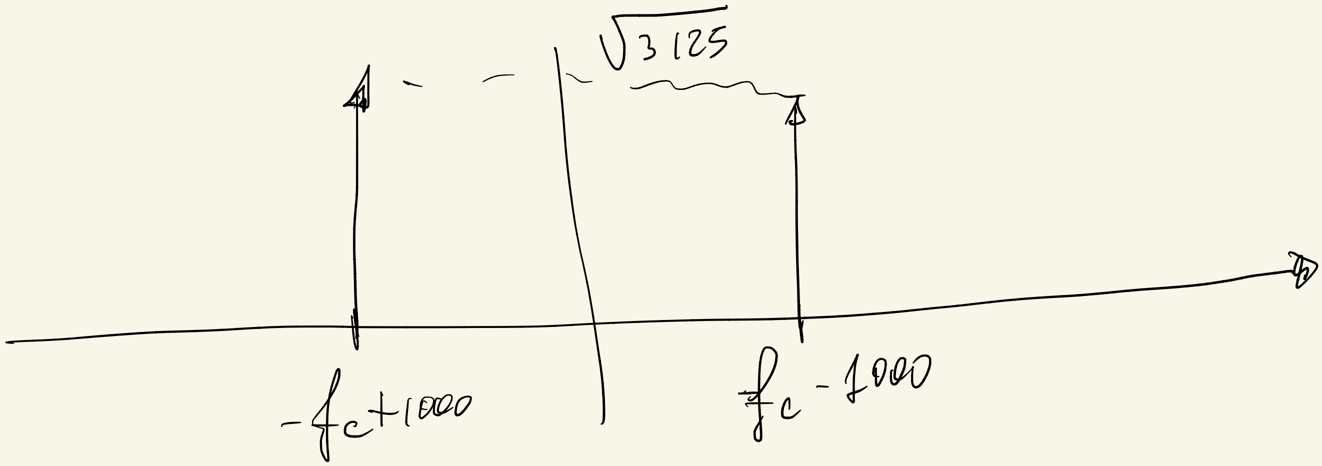
$$25[\delta(f+f_c-f_a) + \delta(f+f_c+f_a) - 2j\,\delta(f+f_c-f_a) + 2j\,\delta(f+f_c+f_a)$$



$$M(f) = 25\left(\delta(f - f_c + 1000) + \delta(f + f_c - 1000)\right)$$
$$+ 50j\left(\delta(f - f_c + 1000) - \delta(f + f_c - 1000)\right) =$$
$$= (25 + 50j)\,\delta(f - f_c + 1000) + (25 - 50j)\,\delta(f + f_c - 1000)$$

the magnitude

$$|M(f)| = \sqrt{(25)^2 + (50)^2}\left(\delta(f - f_c + 1000) + \delta(f + f_c - 1000)\right) =$$
$$+ \delta(f + f_c - 1000))$$
$$= \sqrt{3125}\,(\delta(f - f_c + 1000)$$



$\sqrt{3125}$
$-f_c+1000$
f_c-1000

Ex. 2

$B = 15\,KHz$

(a) $f_s = 2B = 30$ Ksamples/s

(b) $L = \underline{65536} \Rightarrow \underline{\nu} = \log_2 L = 16$

(c) $A = 1 \Rightarrow \boxed{\sigma^2 = \frac{\Delta^2}{12}}$ where $\Delta = \frac{2A}{L}$ $\underline{(-\Delta, \Delta)}$

$\Rightarrow \sigma^2 = \frac{4A^2}{12L^2} = \frac{4}{12(65536)^2}$

(d) $R_b = f_s \cdot \nu = 16 \cdot 30 \cdot 10^3 = \underline{480\,Kbps}$

(e) $\underline{f_s'} = \underline{44\ \text{Ksamples/s}} \Rightarrow \underline{R_b'} = \underline{f_s'} \cdot \underline{\nu} = \underline{704\,kbps}$

$B_{min} = \frac{R_s}{2} = \frac{R_b}{2} = \underline{352\,KHz}$

(f) $B_{min} = \frac{R_s}{2} = \frac{R_b}{2\log_2 M} < \underline{100 \cdot 10^3} \Rightarrow$

$\nu = \log_2 M > \frac{352 \cdot 10^3}{100 \cdot 10^3} = 3{,}5 \Rightarrow M = 16$

$\nu = 4$

for $M = 16 \Rightarrow \underline{\underline{B_{min}}} = \dfrac{R_b}{2 \cdot \log_2 16} = 88\,KHz$

$\Downarrow$

to use the whole bandwidth B

$$B = B_{min}(1+\alpha) = 88 \cdot 10^3 (1+\alpha) = 100 \cdot 10^3$$

$\Downarrow$

$$\boxed{\alpha = 0{,}136}$$

Ex 3

$R_b = 10$ Mbps $\Rightarrow$ $B_{min} = 5$ MHz

Which is EXACTLY the available channel bandwidth

When $B = B_{min}$ I can transmit without ISI ONLY if I use the SINC as a pulse.

BER

antipodal binary $BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

orthogonal binary $BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

ON-OFF signalling $BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

ML

MAP

$\underbrace{\text{GAUSSIAN NOISE}}$

M $\quad p(s_i) = \frac{1}{M} \; \forall i$ $\quad MAP \rightarrow ML \rightarrow$

$\underline{Y}$ vector of gaussian r.v.

$y_i = s_i + n_i$ $\qquad n_i \to \sigma_n^2 = N_0/2$

$\uparrow$

$\psi_i(t)$ $\qquad y_j = s_j + n_j$ $\qquad n_i$ and n_j independent

$$E[y_i y_j] = \phi$$

ML $\qquad \underline{Y}$ maximizes the LIKELYHOOD FUNCTION

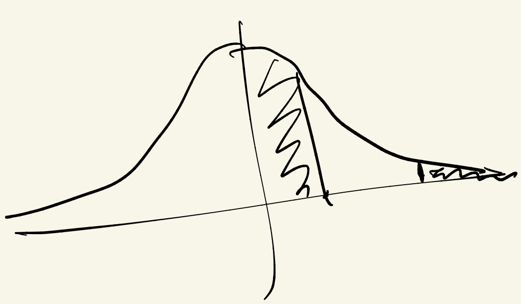
$$\Rightarrow f(\underline{Y}/\underline{s_i}) = \prod_{j=1}^{M} f(y_j/s_i)$$

where
$$f(y_j/s_i) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y_j-s_i)^2}{N_0}}$$

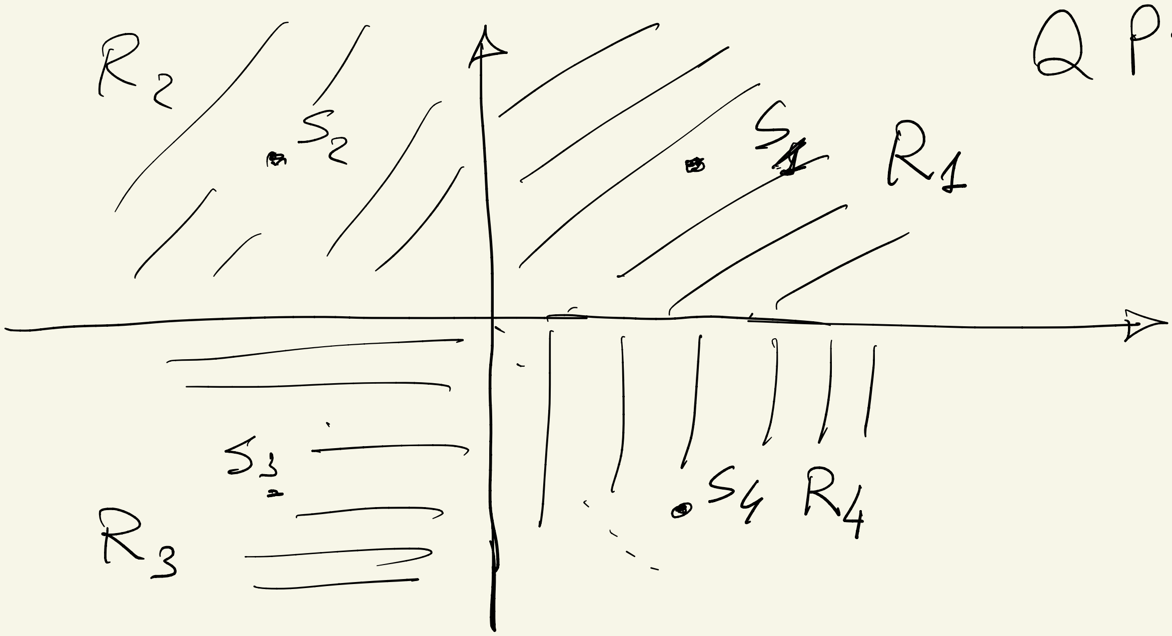
$$\ln[f(\underline{Y}/\underline{s_i})] = -\frac{N}{2}\ln(\pi N_0) - \frac{1}{N_0}\sum_{k=1}^{M}(y_k - s_{ik})^2$$

$$D(\underline{Y}, \underline{s_i}) = \sum_{k=1}^{M}(y_k - s_{ik})^2$$

$\underline{s_i}$ $\quad$ ML $\Rightarrow$ MINIMUM DISTANCE CRITERIA



QPSK



ML DECISION REGIONS

$R_j \qquad \underline{y} \in R_j \longrightarrow s_j$

$$D(\underline{y}, \underline{s}_i) = \sum_{k=1}^{M} y_k^2 - 2 \underbrace{\sum_{k=1}^{M} y_k s_{ik}}_{\parallel} + \sum_{k=1}^{N} s_{ik}^2$$

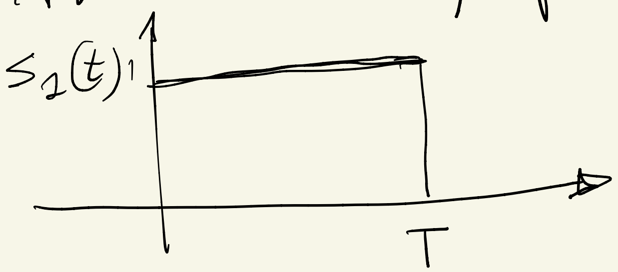
$$\|\underline{Y}\|^2 - 2\,\underline{Y}\cdot\underline{s}_i + \|s_i\|^2$$

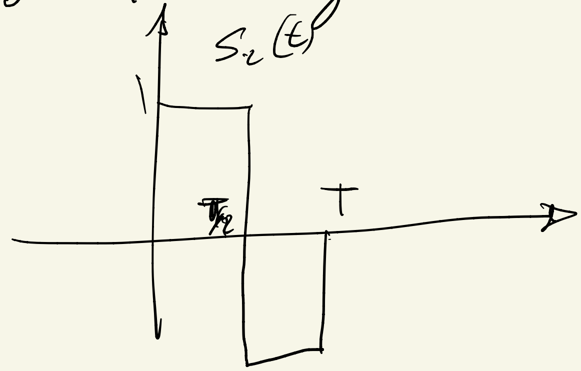
$$D(\underline{y}, \underline{s}_i) \gtrless D(\underline{y}, s_j) \qquad -2\,\underline{Y}\,\underline{s}_i \gtrless -2\underline{Y}\,\underline{s}_j$$

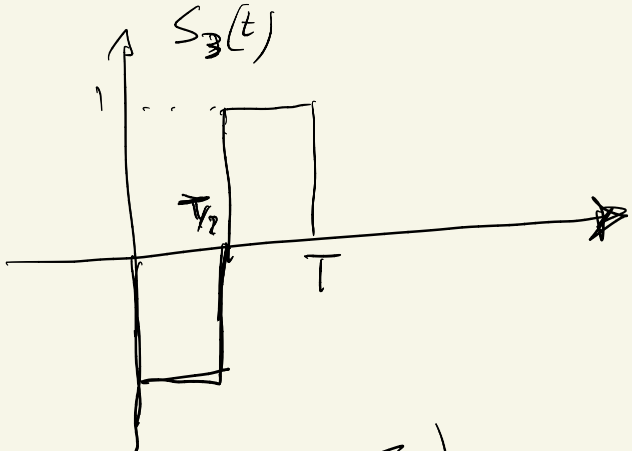
MAXIMUM CORRELATION CRITERIA

EXERCISE

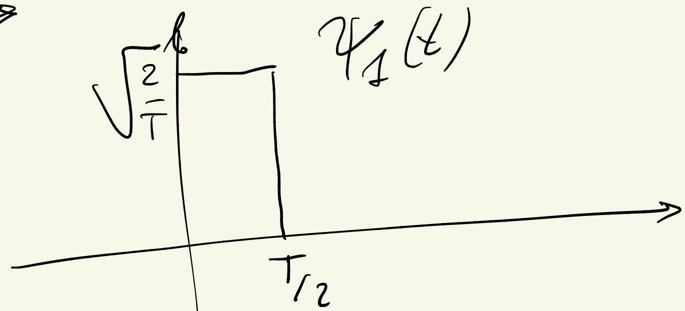
AWGN 3 equiprobable messages m_1, m_2, m_3

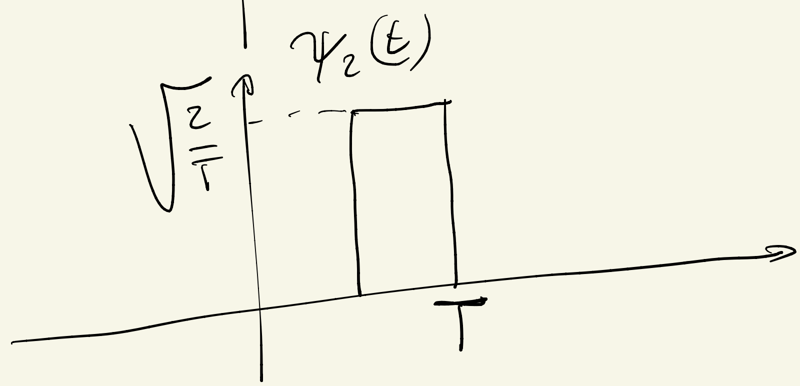






$N = 2$

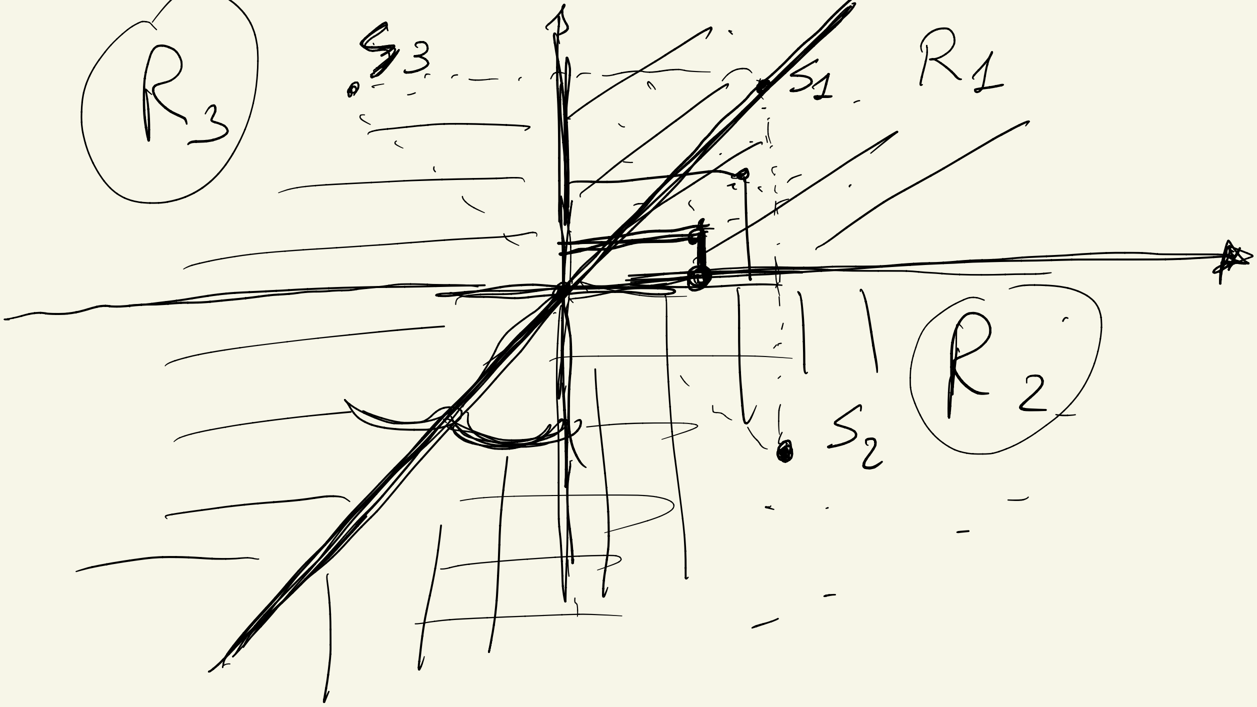




$$\underline{s}_1 = \left(\sqrt{\frac{T}{2}}, \sqrt{\frac{T}{2}}\right)$$

$$\underline{s}_2 = \left(\sqrt{\frac{T}{2}}, -\sqrt{\frac{T}{2}}\right)$$

$$\underline{s}_3 = \left(-\sqrt{\frac{T}{2}}, \sqrt{\frac{T}{2}}\right)$$



$$P_e = \frac{1}{3} P(e/S_1) + \frac{1}{3} P(e/S_2) + \frac{1}{3} P(e/S_3)$$

$P(e/S_1)$ $\quad \underline{y} = ?$ $\quad y_1 = S_{11} + n_1 = \sqrt{\frac{T}{2}} + n_1$

$$y_2 = \sqrt{\frac{T}{2}} + n_2$$

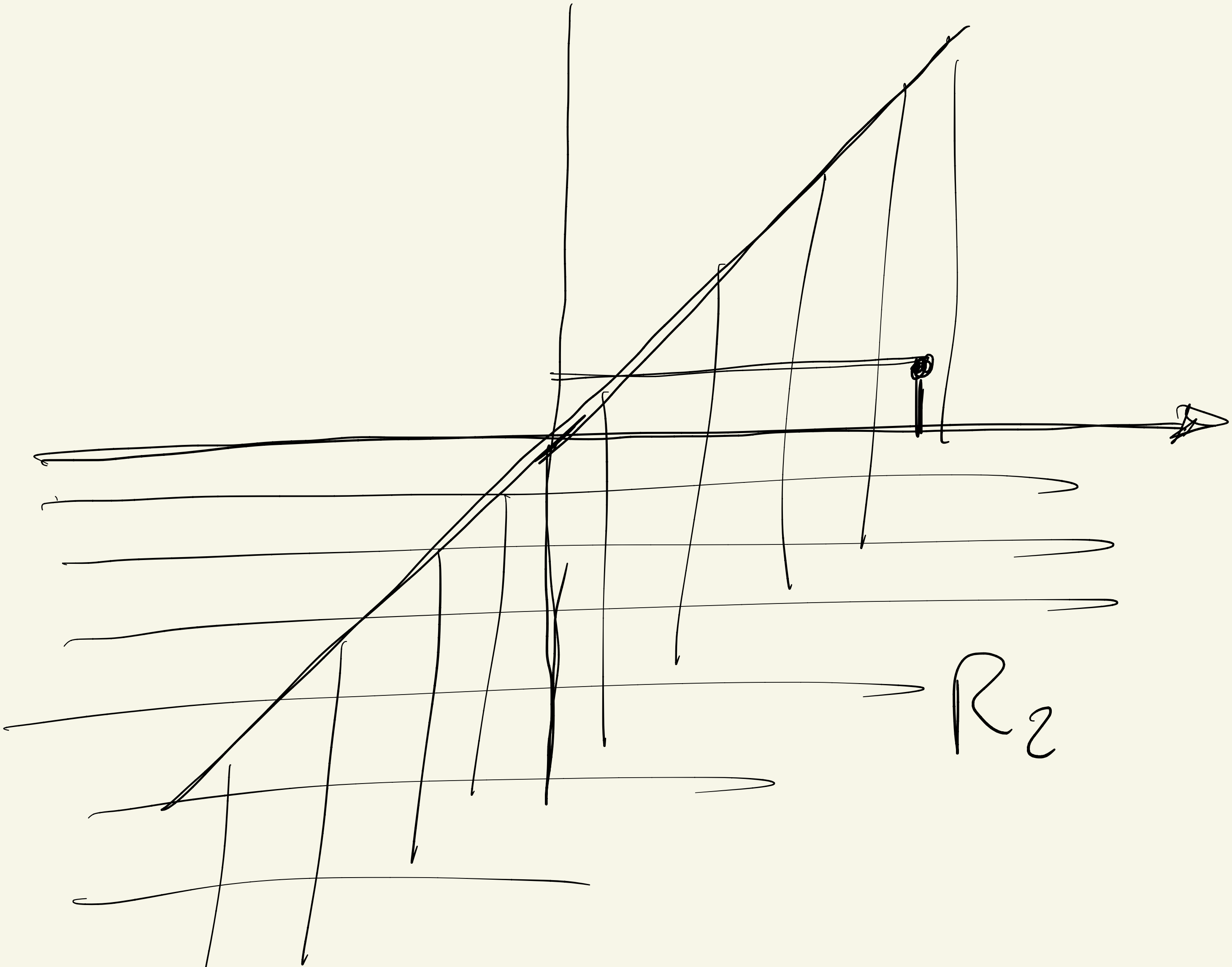
$\underline{y} = (y_1, y_2) \in R_1$

$$\boxed{y_1 > \phi \text{ and } y_2 > \phi}$$

$(y_1 > y_2)$

S_2
$$\boxed{\begin{aligned} y_1 &= \sqrt{\frac{T}{2}} + n_1 \\ y_2 &= -\sqrt{\frac{T}{2}} + n_2 \end{aligned}}$$

$$P_c(S_2) = P(y_2 < \phi / S_2) \cdot P(y_1 > y_2 / S_2)$$



$y_1 > y_2$
$y_2 < \phi$

R_3 $y_1 < y_2$
$y_2 > \phi$

No

$P(y_1 > y_2 / S_2)$

$$P\left(\sqrt{\frac{T}{2}} + n_1 > -\sqrt{\frac{T}{2}} + n_2\right) = P\left(n_1 - n_2 > -2\sqrt{\frac{T}{2}}\right)$$

$P(e/s_1)$

$$P_c = P(c/s_1) = \underbrace{P(y_1 > \phi,\ y_2 > \phi / s_1)}$$

$$\underline{P(e/s_1) = 1 - P_c}$$

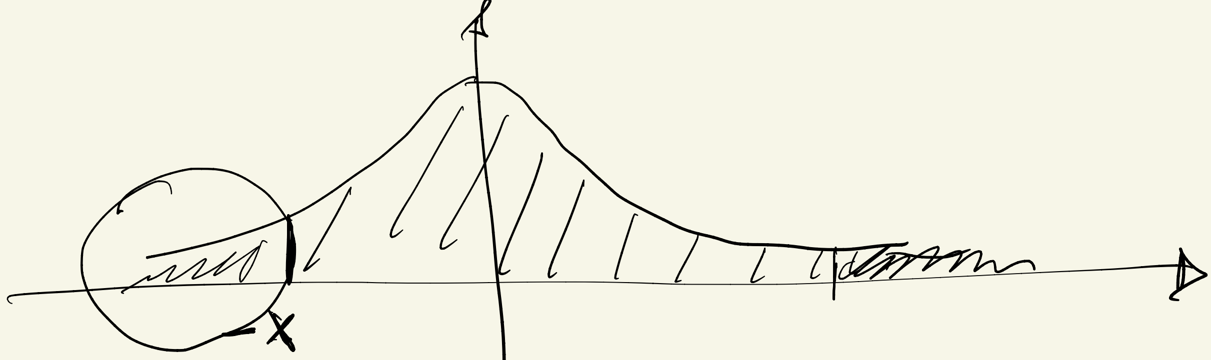
$$P_c = P(y_1 > \phi / s_1) \cdot P(y_2 > \phi / s_1)$$

$$P(y_1 > \phi / s_1) = P\left(\sqrt{\frac{T}{2}} + n_1 > \phi\right) =$$

$$= P\left(n_1 > -\sqrt{\frac{T}{2}}\right) = \frac{1}{\sqrt{\pi N_0}} \int_{-\sqrt{\frac{T}{2}}}^{+\infty} e^{-\frac{x^2}{N_0}} dx =$$

$$\boxed{\frac{x^2}{N_0} = \frac{t^2}{2} \qquad t = \sqrt{\frac{2}{N_0}}\, x}$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_{-\sqrt{\frac{2}{N_0}}\sqrt{\frac{T}{2}}}^{+\infty} e^{-t^2/2} \sqrt{\frac{N_0}{2}}\, dt = \frac{1}{\sqrt{2\pi}} \int_{-\sqrt{\frac{T}{N_0}}}^{+\infty} e^{-t^2/2}\, dt =$$



$$= 1 - Q\left(\sqrt{\frac{T}{N_0}}\right)$$

$$P(y_2 > \phi / s_1) = 1 - Q\left(\sqrt{\frac{T}{N_0}}\right)$$

$$P(e/s_1) = 1 - P_c = 1 - \left(1 - Q\left(\sqrt{\frac{T}{N_0}}\right)\right)^2 =$$

$$= \cancel{1} - \cancel{1} + 2Q\underbrace{\left(\sqrt{\frac{T}{N_0}}\right)}_{10^{-4}} - \underbrace{\cancel{Q^2\left(\sqrt{\frac{T}{N_0}}\right)}}_{10^{-8}}$$

$$P(e/s_1) \approx 2Q\left(\sqrt{\frac{T}{N_0}}\right)$$

$$\underbrace{P(e/s_2)} = P(e/s_3) \neq P(e/s_1)$$

$$P(e/s_2) = 1 - P_c(s_2)$$